

OUTP-97-68P  
 UA-NPPS 8/97  
 cond-mat/9711257

# N=1 Supersymmetric Spin-Charge Separation in effective gauge theories of planar magnetic superconductors

G.A. Diamandis<sup>a</sup>, B.C. Georgalas<sup>a</sup> and N.E. Mavromatos<sup>\*</sup>

University of Oxford, Department of (Theoretical) Physics, 1 Keble Road OX1 3NP, Oxford, U.K.

## Abstract

We present a  $N = 1$  Supersymmetric extension of a spin-charge separated effective  $SU(2) \times U_S(1)$  ‘particle-hole’ gauge theory of excitations about the nodes of the gap of a  $d$ -wave planar magnetic superconductor. The supersymmetry is achieved without introducing extra degrees of freedom, as compared to the non-supersymmetric models. The only exception, the introduction of gaugino fields, finds a natural physical interpretation as describing interlayer coupling in the statistical model. The low-energy continuum theory is described by a relativistic (2+1)-dimensional supersymmetric  $CP^1$   $\sigma$ -model with Gross-Neveu-Thirring-type four-fermion interactions. We emphasize the crucial rôle of the  $CP^1$  constraint in inducing a non-trivial dynamical mass generation for fermions (and thus superconductivity), in a way compatible with manifest  $N = 1$  supersymmetry. We also give a preliminary discussion of non-perturbative effects. We argue that supersymmetry suppresses the dangerous for superconductivity instanton contributions to the mass of the perturbatively massless gauge boson of the unbroken  $U(1)$  subgroup of  $SU(2)$ . Finally, we point out the possibility of applying these ideas to effective gauge models of spin-charge separation in one-space dimensional superconducting chains of holons, which, for example, have recently been claimed to be important in the stripe phase of underdoped cuprates.

November 1997

<sup>a</sup> On leave from University of Athens, Physics Department, Elementary Particle and Nuclear Physics Section, Athens GR-157 71, Greece.

(\*) P.P.A.R.C. Advanced Fellow.

# 1 Introduction

In ref. [1] it was argued that the doped large- $U$  Hubbard (antiferromagnetic) models possess a *hidden* local *non-Abelian*  $SU(2) \times U_S(1)$  phase symmetry related to spin interactions. This symmetry was discovered using an appropriate ‘particle-hole symmetric formalism’ for the electron operators [2], and employing a generalized *slave-fermion* ansatz for *spin-charge separation*, which allows intersublattice hopping for holons, viewed as fermions. The spin-charge separation may be physically interpreted as implying an effective ‘substructure’ of the electrons due to the many body interactions in the medium. This sort of idea, originating from Anderson’s RVB theory of spinons and holons [3], was also pursued recently by Laughlin, although from a (formally at least) different perspective [4].

In ref. [1] we have argued in favour of the opening of a fermion gap at the nodes of a  $d$ -wave gap of a superconducting antiferromagnet. Linearization of the fermion spectrum about such nodes leads to a relativistic Dirac spectrum for holons, with the rôle of the limiting velocity being played by the fermi velocity [5, 1]. Such systems might be of relevance to the physics of high-temperature superconductors, since recently it is believed that high-temperature superconductivity in cuprates is highly anisotropic and the gap symmetry is of  $d$ -wave type [6], with the gap vanishing along lines of *nodes* on the Fermi surface <sup>1</sup>.

The key suggestion in ref. [1], which lead to the non-abelian gauge symmetry structure for the doped antiferromagnet, with the constraint of *not more than one electron per lattice site*, was the *slave-fermion* spin-charge separation ansatz for physical electron operators [1]:

$$\chi_{\alpha\beta,i} \equiv \begin{pmatrix} c_1 & c_2 \\ c_2^\dagger & -c_1^\dagger \end{pmatrix}_i \equiv \hat{\psi}_{\alpha\gamma,i} \hat{z}_{\gamma\beta,i} = \begin{pmatrix} \psi_1 & \psi_2 \\ -\psi_2^\dagger & \psi_1^\dagger \end{pmatrix}_i \begin{pmatrix} z_1 & -\bar{z}_2 \\ z_2 & \bar{z}_1 \end{pmatrix}_i \quad (1)$$

where  $i$  is a *lattice site* index,  $c_\alpha$ ,  $\alpha = 1, 2$  are electron annihilation operators, the Grassmann variables  $\psi_i$ ,  $i = 1, 2$  play the rôle of holon excitations, while the bosonic fields  $z_i$ ,  $i = 1, 2$ , represent magnon (bosonized spinon) excitations [3]. The ansatz (1) has spin-electric-charge separation, since only the fields  $\psi_i$  carry *electric* charge.

As argued in ref. [1] the ansatz is characterised by the following *hidden local phase* (gauge) symmetry structure:

$$G = SU(2) \times U_S(1) \times U_E(1) \quad (2)$$

The gauge  $SU(2)$  symmetry pertains to the spin degrees of freedom. The local  $U_S(1)$  ‘statistical’ phase symmetry allows fractional statistics of the spin and charge excitations. This is an exclusive feature of the three dimensional geometry, and is similar in spirit to the bosonization technique of the spin-charge separation ansatz of ref. [11]. Finally the  $U_E(1)$  symmetry is due to the electric charge of the holons.

---

<sup>1</sup>There is also recent experimental evidence on the possibility of the opening of a gap at such nodes, triggered by either magnetic fields [7] or by magnetic impurities [8], and although such phenomena might admit alternative (more conventional) explanations [9, 10], however the rôle of spin-charge separation in this context still remains a challenging project.

It is the purpose of this work to discuss the possibility of a *hidden supersymmetry* in the ansatz (1). Note that supersymmetric extensions of  $J = \pm 2t$  models for doped antiferromagnets, in one and two spatial dimensions, have already appeared in the existing literature [12], even in the context of spin-charge separated anyon models [13]. However, as far as we are aware, such supersymmetries have not been associated so far with any specific dynamical properties of the antiferromagnet. In contrast, in our approach here, based on the non-trivial ansatz (1), the supersymmetry constitutes a non-trivial *dynamical* property of the spin-charge separated vacuum for *holons* and *spinons*, by viewing them as *supersymmetric partners*. Due to the rich group structure (2), many possibilities arise in the study of the phase diagrams of these theories, in the context of the modern perspective advocated in the work of Seiberg and Witten [14, 15]. In particular, duality symmetries in the infrared region of the supersymmetric model, connecting various theories with the same non-trivial infrared fixed-point [15], may prove very useful in a renormalization-group study of the dynamics of the gauge fields in both, the superconducting and the normal phases of the model, in the spirit of ref. [16]. The important issue is that the introduction of  $N = 1$  supersymmetry, *hidden* in the spin-charge separation ansatz (1), does not require the introduction of unphysical degrees of freedom. As we shall see, the only extra degrees of freedom, as compared to the non-supersymmetric case [1], are the gauginos of the local hidden gauge symmetry, which, however, admit the natural interpretation of describing *interlayer* hopping of spin *and* charge degrees of freedom (hopping of ‘real’ electrons).

We should stress that, within a condensed-matter context, the supersymmetry refers to the relativistic field theory at the nodes of a  $d$ -wave superconducting gap <sup>2</sup>. In this sense, the supersymmetric dynamics of the spinons and holons would require equality of the spin gap with the fermion (superconducting) gap at such nodes. At a microscopic level, this would imply some particular relation among the microscopic parameters of the model, such as hopping matrix elements and Heisenberg interactions. This calls for comparison with the  $J = \pm 2t$  special point, where the graded (supersymmetric) algebra in the spectrum of the doped antiferromagnets appears [12, 13]. However, as we shall see, the situation in our case is more complicated, since there are more parameters entering the dynamical scenario of the gauge theory based on the spin-charge separation ansatz (1).

## 2 Review of the (continuum) model and its superconducting properties

Before embarking to a description of the supersymmetric extension we consider it as useful to review first the properties of the statistical model of ref. [1], some of which will be crucial for the supersymmetric extension. The pertinent long-wavelength gauge model, describing the low-energy dynamics of the large- $U$  Hubbard antiferromagnet in the spin-charge separation phase (1), can be cast in a conventional relativistic lattice gauge-theory, provided one changes

---

<sup>2</sup>Galilean supersymmetry, as symmetry of the spectrum between bosonic and fermionic degrees of freedom, may also occur away from the nodes. This is left for future work.

representation of the  $SU(2)$  group, and, instead of working with  $2 \times 2$  matrices, one uses a representation in which the fermionic matrices  $\hat{\psi}_{\alpha\beta}$  are represented as two-component (Dirac) spinors in ‘colour’ space:

$$\tilde{\Psi}_{1,i}^\dagger = \begin{pmatrix} \psi_1 & -\psi_2^\dagger \end{pmatrix}_i, \quad \tilde{\Psi}_{2,i}^\dagger = \begin{pmatrix} \psi_2 & \psi_1^\dagger \end{pmatrix}_i, \quad i = \text{Lattice site} \quad (3)$$

By assuming a background  $U_S(1)$  field of flux  $\pi$  per lattice plaquette [5], and considering quantum fluctuations around this background for the  $U_S(1)$  gauge field, one can obtain the conventional lattice Dirac action for the fermion excitations about a node in the fermi surface [5, 17, 1].

In the above context, a strongly coupled  $U_S(1)$  group can dynamically generate a mass gap in the holon spectrum [18, 5, 19, 20, 21], which breaks the  $SU(2)$  local symmetry down to its Abelian subgroup generated by the  $\sigma_3$  Pauli matrix [1, 22]. From the view point of the statistical model of ref. [1], the breaking of the  $SU(2)$  symmetry may be interpreted as restricting the holon hopping effectively to a single sublattice, since the intersublattice hopping is suppressed by the mass of the gauge bosons.

The (naive) continuum limit of the low-energy theory about such nodes on the fermi surface of the planar antiferromagnet, then, is described by a  $CP^1$  model coupled to Dirac fermions [5, 1]:

$$\mathcal{L}_2 = g_1^2 |(\partial_\mu - (g_2/g_1)B_\mu^a \sigma^a - a_\mu)z|^2 + i\bar{\Psi}D_\mu\gamma_\mu\Psi \quad (4)$$

where now  $D_\mu = \partial_\mu - ia_\mu - i(g_2/g_1)\sigma^a B_{a,\mu} - \frac{e}{c}A_\mu$ ,  $g_i^2$ ,  $i = 1, 2$  have dimensions of mass,  $B_\mu^a$  is the gauge potential of the local (‘spin’)  $SU(2)$  group, generated (in two-component notation for fermions) by the Pauli matrices  $\sigma^a$ ,  $a_\mu$  the  $U_S(1)$  (‘fractional statistics’) field, and  $A_\mu$  is an external electromagnetic potential, which will be ignored in the subsequent discussion. In terms of the microscopic model,  $g_1^2 \sim J\delta$ , where  $J$  is the Heisenberg exchange energy, and  $\delta$  is the doping concentration. An important ingredient in the above formalism is the no-double occupancy constraint, which in terms of the  $z$  and  $\Psi_\alpha$ ,  $\alpha = 1, 2$ , fields, with  $\alpha$  a ‘colour’ index, can be written as:

$$\sum_{\alpha=1}^2 [\bar{z}^\alpha z_\alpha + \beta \bar{\Psi}^\alpha \sigma_3 \Psi_\alpha] = 1 \quad (5)$$

where  $\sigma_3$  acts in spinor space, and the fermions  $\Psi$  are viewed as *two-component* spinors, related to the spinors  $\tilde{\Psi}$  (3) by appropriate rescalings so as to ensure the canonical kinetic (Dirac) term<sup>3</sup>. This results in the presence of the constant  $\beta$  (with dimensions of  $[\text{mass}]^{-2}$ ) in the constraint (5) [17]. In the context of the microscopic model, these constants are expressed in terms of the hopping and Heisenberg exchange energies [17, 1], and one has that  $|\beta| \ll 1$ . It can be shown [1] that the constraint (5) is essential in ensuring the consistency of the ansatz (1) with the canonical commutation relations of the electron operators.

The presence of the  $\Psi^\dagger\Psi$  (non-relativistic) fermion number term in the constraint (5) appears at first sight to complicate things, since the conventional  $CP^1$  constraint  $|z|^2 = 1$  is no longer

---

<sup>3</sup>In the model of ref. [1], due to the Dirac nature of the resulting spinors,  $\Psi^\dagger$  and  $\Psi$  are viewed as independent variables in a path integral, which implies that one can redefine  $\Psi^\dagger \rightarrow \bar{\Psi}$ .

valid. However, these extra terms can be rendered innocuous for the dynamics of the effective theory. Indeed, by integrating out the (non-propagating) gauge fields in (4) we obtain [23]:

$$\begin{aligned}\mathcal{L}_B = & g_1^2 \partial^\mu \bar{z}^\alpha \partial_\mu z_\alpha + i \bar{\Psi}^\alpha \not{\partial} \Psi_\alpha + \\ & \frac{g_1^2}{2} [1 - \beta \bar{\Psi}^\alpha \sigma_3 \Psi_\alpha]^{-1} \text{Tr} \left( \bar{z}^\alpha \partial_\mu z_\alpha - z_\alpha \partial_\mu \bar{z}^\alpha - i g_1^{-2} \bar{\Psi}^\alpha \gamma_\mu (1 + \sigma^a) \Psi_\alpha \right)^2 + \\ & 6 \ln[1 - \beta \bar{\Psi}^\alpha \sigma_3 \Psi_\alpha]\end{aligned}\quad (6)$$

where the last term is absent in the usual  $CP^1$  models. Expanding this term in powers of the (small) parameter  $\beta \ll 1$ , one obtains:

$$6 \ln[1 - \beta \bar{\Psi}^\alpha \sigma_3 \Psi_\alpha] \simeq -6\beta \bar{\Psi}^\alpha \sigma_3 \Psi_\alpha + 3\beta^2 (\bar{\Psi}^\alpha \sigma_3 \Psi_\alpha)^2 + \dots \quad (7)$$

where the  $\dots$  indicate six- and higher order -fermion contact terms, not renormalizable, even in large-N limits, which constitute irrelevant operators, in a renormalization-group sense, not affecting the low-energy (infrared) structure of the effective theory, we are interested in.

Applying a Hartree-Fock linearization to the four-fermion interactions, one obtains terms of the form:

$$3\beta^2 < \bar{\Psi}^\beta \sigma_3 \Psi_\beta > \bar{\Psi}^\alpha \sigma_3 \Psi_\alpha \quad (8)$$

Collecting the  $\Psi^\dagger \Psi$  terms together, one then obtains a fermion-density term in the effective lagrangian of the form:

$$L_\mu = (-6\beta + 3\beta^2 < \bar{\Psi}^\alpha \sigma_3 \Psi_\alpha >) \bar{\Psi}^\beta \sigma_3 \Psi_\beta \quad (9)$$

Upon inserting the constraint (5) via a Lagrange multiplier field  $\lambda(x)$  in the path integral, one may expand [24] about the vacuum defined by  $< \lambda(x) > \propto m_Z^2 \neq 0$ , where  $m_Z$  is a spinon gap (magnon mass), in appropriate units. Then, we can tune the parameter of our system so as to define a fully relativistic field theory about the nodes of a  $d$ -wave gap [1], such that, when  $\beta \neq 0$ , the following is satisfied:

$$< \lambda(x) > \beta - 6\beta + 3\beta^2 < \bar{\Psi}^\alpha \sigma_3 \Psi_\alpha > = 0 \quad (10)$$

Note that the non-zero dynamical condensate of the (non-relativistic) operator  $< \bar{\Psi}^\alpha \sigma_3 \Psi_\alpha >$ , obtained above, is compatible with a dynamical opening of a fermion mass gap in the resulting relativistic field theory.

In this way, the fermion terms in the constraint (5) decouple, and the effective theory of the excitations at the nodes of the  $d$ -wave superconducting gap can be described, up to terms that are renormalization-group irrelevant operators in the infrared, by the effective theory (6) with Thirring four-fermion interactions, and a standard  $CP^1$  constraint:

$$\sum_{\alpha=1}^2 |z^\alpha|^2 = 1 \quad (11)$$

The latter implies that the magnon fields, in their massive (spin gap) phase, can be integrated out in a standard fashion in the path integral [24], to yield an alternative low-energy theory, that of a dynamical  $SU(2) \times U_S(1)$  gauge group, with Maxwell kinetic terms for the gauge fields, which are the dominant terms in a derivative expansion.

Superconductivity in this model occurs [1] as a result of dynamical generation of a parity-conserving fermion mass in the strong-coupling regime of the  $U_S(1)$  gauge field [18, 5, 1], upon coupling the system to external electromagnetic potentials. This dynamical generation phenomenon occurs in the infrared region of the effective theory obtained after  $z$ -magnon integration. In such a theory, upon the opening of a mass gap in the fermion (holon) spectrum, the Feynman matrix element:  $\mathcal{S}^a = \langle B_\mu^a | J_\nu | 0 \rangle$ ,  $a = 1, 2, 3$ , with  $J_\mu = \bar{\Psi} \gamma_\mu \Psi$  the fermion-number current, is non-trivial. Due to the colour-group structure, only the massless  $B_\mu^3$  gauge boson of the  $SU(2)$  group, corresponding to the  $\sigma_3$  generator in two-component notation, contributes to the matrix element. The non-trivial result for  $\mathcal{S}^3$  arises from an *anomalous one-loop graph*, depicted in figure 1, and it is given by [25, 5]:

$$\mathcal{S}^3 = \langle B_\mu^3 | J_\nu | 0 \rangle = (\text{sgn} m_f) \epsilon_{\mu\nu\rho} \frac{p_\rho}{\sqrt{p_0}} \quad (12)$$

where  $m_f$  is the parity-conserving fermion mass, generated dynamically by the  $U_S(1)$  group. As with the other Adler-Bell-Jackiw anomalous graphs in gauge theories, the one-loop result (12) is *exact* and receives no contributions from higher loops [25].

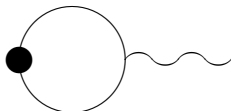


Figure 1: *Anomalous one-loop Feynman matrix element, leading to a Kosterlitz-Thouless-like breaking of the electromagnetic  $U_E(1)$  symmetry, and thus superconductivity, once a fermion mass gap opens up. The wavy line represents the  $SU(2)$  gauge boson  $B_\mu^3$ , which remains massless, while the blob denotes an insertion of the fermion-number current  $J_\mu = \bar{\Psi} \gamma_\mu \Psi$ . Continuous lines represent fermions.*

This unconventional symmetry breaking (12), does *not* have a local order parameter [25, 5], since the latter is inflicted by strong phase fluctuations, thereby resembling the Kosterlitz-

Thouless mode of symmetry breaking<sup>4</sup>. The *massless* gauge boson  $B_\mu^3$  of the unbroken  $\sigma_3 - U(1)$  subgroup of  $SU(2)$  is responsible for the appearance of a *massless pole* in the electric current-current correlator [5], which is the characteristic feature of any *superconducting theory*. As discussed in ref. [5], all the standard properties of a superconductor, such as the Meissner effect, infinite conductivity, flux quantization, London action etc. are recovered in such a case. The field  $B_\mu^3$ , or rather its *dual*  $\phi$  defined by  $\partial_\mu \phi \equiv \epsilon_{\mu\nu\rho} \partial_\nu B_\rho^3$ , can be identified with the Goldstone boson of the broken  $U_{em}(1)$  (electromagnetic) symmetry [5]. We shall come back to the exactness of this result, upon including non-perturbative effects (instantons), in the context of our supersymmetric model, later on.

### 3 N=1 Supersymmetric Gauge Theory of Spin-Charge Separation

We are now ready to discuss the possibility of the emergence of a  $N = 1$  space-time supersymmetry in the ansatz (1). The main idea behind such a supersymmetrization is to view the magnons  $z$  as *supersymmetric partners* of the holons  $\Psi$ . For simplicity, in this note we shall turn off the  $SU(2)$  interactions in (4), keeping only  $U_S(1)$ , which is mainly responsible for the chiral symmetry breaking (mass generation) phenomenon. The incorporation of the gauge group  $SU(2) \times U_S(1)$  (2) is straightforward. In this section we shall demonstrate the possibility of a  $N = 1$ -supersymmetric extension of the action (4), and of the constraint (5), in the absence of (non-supersymmetric) external electromagnetic potentials.

The basic “matter” multiplet of  $N=1$  supersymmetry in three space-time dimensions, can be written in terms of a scalar superfield as [26]

$$\Phi = \phi + \bar{\theta}\chi + \frac{1}{2}\bar{\theta}\theta F \quad (13)$$

which contains a real scalar field,  $\phi$ , a Majorana spinor  $\chi_\alpha$  and a real auxiliary field  $F$ . We consider complex superfields

$$Z = \frac{1}{\sqrt{2}}(\Phi_1 + i\Phi_2) = z + \bar{\theta}\psi + \frac{1}{2}\bar{\theta}\theta F \quad (14)$$

which contain a complex scalar,  $z = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$ , a Dirac spinor,  $\Psi = \frac{1}{\sqrt{2}}(\psi_1 + i\psi_2)$ , and a complex auxiliary field,  $F = \frac{1}{\sqrt{2}}(F_1 + iF_2)$ . The supersymmetry transformations read,

$$\begin{aligned} \delta_S z &= \bar{\xi}\psi \\ \delta_S \psi &= -i\gamma^\mu \xi \partial_\mu \phi + \xi F \\ \delta_S F &= -i\bar{\xi}\not{\partial}\psi \end{aligned} \quad (15)$$

---

<sup>4</sup>This may be important from a condensed-matter viewpoint, since the absence of a local order parameter implies that the opening of a fermion mass gap at the *nodes* of the original *d*-wave superconducting gap of the cuprate does not affect the *d*-wave nature of the state.

and the supersymmetric invariant lagrangian is given by the highest component  $(\bar{\theta}\theta)$  of the superfield

$$\bar{D}Z^*DZ \quad (16)$$

where

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} - i(\not{\partial}\theta)_\alpha \quad (17)$$

is the supersymmetry covariant derivative.

The gauge field is incorporated in a real spinor superfield which, in the Wess-Zumino gauge, takes the form

$$V_\alpha = i(\not{a}\theta)_\alpha + \frac{1}{2}\bar{\theta}\theta\eta_\alpha \quad (18)$$

where  $\eta$  is the supersymmetric partner of the gauge field (gaugino).

The supersymmetric gauge invariant lagrangian for the matter fields which in terms of superfields is the highest component of the superfield

$$\bar{\mathcal{D}}Z^*\mathcal{D}Z \quad (19)$$

with

$$\mathcal{D}_\alpha = D_\alpha - iV_\alpha \quad (20)$$

In terms of component fields the lagrangian reads:

$$L = g_1^2[D_\mu \bar{z}^\alpha D^\mu z^\alpha + i\bar{\Psi}\not{D}\Psi + \bar{F}^\alpha F_\alpha + 2i(\bar{\eta}\Psi^\alpha \bar{z}^\alpha - \bar{\Psi}^\alpha \eta z^\alpha)] \quad (21)$$

where  $D_\mu$  denotes the gauge covariant derivative with respect to the  $U_S(1)$  field, and for convenience we have rescaled the fermion fields  $\Psi$  and the auxiliary field  $F$  by  $g_1$ , as compared to the non-supersymmetric case, in order to facilitate our superfield formalism. Notice that (21) contains a supersymmetric partner of the statistical gauge field  $U_S(1)$ . From the point of view of the ansatz (1), this is the defining property of the  $N = 1$  supersymmetric point of the model, in the sense that the gauge interaction  $U_S(1)$  ‘doubles’ its degrees of freedom. From the point of view of the statistical model of ref. [1], this doubling will only be reflected in the form of the effective action, after integrating out the  $U_S(1)$  field. As explained in ref. [1], this field is responsible for yielding fractional statistics to the holons and spinons in three dimensions, and as such should be integrated out in the effective action of the physically observable degrees of freedom.

It is important to notice that the constraint (5) admits a  $N = 1$  supersymmetric formulation, in terms of the superfields  $Z^\alpha$  (14):

$$\sum_{\alpha=1}^2 \bar{Z}^\alpha Z_\alpha = 1 \quad (22)$$

Upon integrating out the (non-propagating)  $a_\mu$  and gaugino  $\eta$  fields in a path integral for the lagrangian (21), and using the constraint (22), it is immediate to obtain the following effective



action of holons and spinons in the spin-charge separation ansatz (1) at the supersymmetric point:

$$\begin{aligned} \mathcal{L}_S = & g_1^2 [\partial^\mu \bar{z}^\alpha \partial_\mu z_\alpha + i \bar{\Psi}^\alpha \not{\partial} \Psi_\alpha + \bar{F}^\alpha F_\alpha + \frac{1}{2} (\bar{z}^\alpha \partial_\mu z_\alpha - z_\alpha \partial_\mu \bar{z}^\alpha - i \bar{\Psi}^\alpha \gamma_\mu \Psi_\alpha)^2] + \\ & g_1^6 \ln \left( g_1^{-2} \sum_\alpha \{ |z_\alpha|^2 \bar{\Psi}^\alpha \Psi_\alpha + \sum_{\beta \neq \alpha} z_\alpha \bar{z}^\beta \bar{\Psi}^\alpha \Psi_\beta + \frac{1}{2} [z^\alpha z_\alpha \bar{\Psi}^\alpha \Psi_\alpha^* + \sum_{\beta \neq \alpha} z^\alpha z^\beta \bar{\Psi}_\alpha \Psi_\beta^* + h.c.] \} \right) \end{aligned} \quad (23)$$

The auxiliary fields  $F_\alpha$  can be solved by means of the constraint (22):

$$\bar{F}^\alpha F_\alpha = \frac{1}{2} (\bar{\Psi}^\alpha \Psi_\alpha)^2 \quad (24)$$

The terms inside the logarithm in (23) contain no bare mass terms, but only interaction terms among  $z$  and  $\Psi$  fields. This can be readily seen by the following formal expansion:

$$\ln x = 2 \sum_{k=1}^{\infty} \frac{1}{2k-1} \left( \frac{x-1}{x+1} \right)^{2k-1} \quad (25)$$

which truncates due to the Grassman structures in  $x$ .

Several important comments are now in order. First, notice that the supersymmetric extension of the effective lagrangian for spinons and holons contains *both* Gross-Neveu and Thirring four-fermion interactions. This can be seen by using the Fierz rearrangement formula in three space-time dimensions:

$$\gamma_{ab}^\mu \gamma_{\mu,cd} = 2\delta_{ad}\delta_{bc} - \delta_{ab}\delta_{cd} \quad (26)$$

upon which the four fermion Thirring interactions become:

$$[\bar{\Psi}_\alpha \gamma_\mu \Psi_\alpha]^2 = -3 \sum_{\alpha=1}^2 [\bar{\Psi}_\alpha \Psi_\alpha]^2 - 2[\bar{\Psi}_1 \Psi_1 \bar{\Psi}_2 \Psi_2 + 2\bar{\Psi}_1 \Psi_2 \bar{\Psi}_2 \Psi_1] \quad (27)$$

showing that the Gross-Neveu terms in the Thirring interactions cannot cancel the ones appearing in (23) due to the supersymmetric extension.

As a result of supersymmetry, the couplings of the four-fermion terms are all related, and are of order  $g_1^2$ . In the context of the statistical model, such a restriction will imply special relations among the microscopic parameters, such as hopping elements, Heisenberg exchange energies, doping concentration etc. For instance, in the special case of ref. [17], where the next-to-nearest-neighbour (NNN) hopping element  $t'$  is assumed dominant, with  $t \sim 0$ , one can show that four-fermion Gross-Neveu type terms come with generic coefficients of order  $(t')^2/(J'\delta^2)$ , with  $J'$  ( $\ll J$ ) the NNN Heisenberg exchange energy. In such a situation, supersymmetry enforces the relation  $t' \sim \sqrt{JJ'}\delta^{3/2}$ , which may be interpreted as implying that supersymmetric points in our formalism may be obtained by tuning the doping concentration. Such restrictions may be compatible with the  $t' = J'/2$  supersymmetric point of ref. [12, 13]). In the case above,

such an extra restriction implies underdoped situation  $\delta^{3/2} \sim \sqrt{J'/4J} \ll 1$ . In more realistic models, like the one discussed in ref. [1], involving nearest-neighbour hopping, there will be more constraints, involving the hopping element  $t$ , etc. A complete analysis along these lines falls beyond our present scope.

Another important issue concerns the physical interpretation of the Majorana fermion  $\eta$ , which, as one can see from (21), (23), leads to effective electric-charge violating interactions on the spatial planes. From our two-spatial dimension point of view, such violations may admit the interpretation of describing *interlayer* hopping of spin and charge degrees of freedom (hopping of real four-space-time-dimensional electrons). In this interpretation, the gaugino  $\eta$  terms in (21), constitute a Majorana-spinor representation of the *absence* of spin *and* charge at a site of the planar lattice system:

$$\int d\eta e^{2i \int d^3x \bar{\eta} \Psi^\alpha z_\alpha + H.C.} \quad (28)$$

The reader is advised to draw a comparison with the Grassmann  $\chi, \chi^\dagger$ , representation of a Wilson line ('missing spin' S) in the treatment of *static holes* in refs. [27, 5]:

$$\int d\chi^\dagger d\chi e^{-iS \int dt \sum_i (-1)^i \chi_i^\dagger \chi_i a_0(i,t)} \quad (29)$$

where  $a_0$  is the temporal component of the gauge potential of the  $CP^1$   $\sigma$ -model, describing spin excitations in the antiferromagnet. From this point of view, the existence of  $N = 1$  supersymmetry in the doped antiferromagnets necessitates *interplanar couplings*, through hopping of spin and charge degrees of freedom (electrons) across the planes.

## 4 Dynamical Mass Generation and N=1 Supersymmetry

Next, we proceed to discuss the dynamical scenario for fermion mass generation. First, we note that dynamical-mass generation (pairing) in non-supersymmetric models, with combined Gross-Neveu and Thirring four-fermion interactions, is possible in three space-time dimensions. By using a four-component fermion formalism one obtains consistent solutions to the Schwinger-Dyson (SD) equations, with non-zero mass, which conserve parity and time-reversal invariance [28, 29]<sup>5</sup>. In ref. [29] it was shown that in models with mixed Thirring and Gross-Neveu interactions, it is essentially the Gross-Neveu coupling  $g_{GN}$  which determines the critical behaviour (critical flavour number) of the theory, in a large N expansion. For  $g_{GN} > g_{GN}^c$ ,

---

<sup>5</sup>Note that theories with four-fermion interactions are not in general vector-like, and hence the theorems of ref. [30], for absence of spontaneous violation of parity and time-reversal symmetry due to energetics, cannot apply. However, in our superconducting model, integrating out the magnon fields one obtains [24, 23] a dynamical gauge theory in the infrared. It is in this sense that we are interested only in parity-conserving mass gaps, which from the point of view of the (low-energy) effective gauge theory, are the energetically preferable configurations [30].

where  $g_{GN}^c$  is the critical coupling of the (2+1)-dimensional Gross-Neveu model [31], the system is dominated by the Gross Neveu interaction, while for  $g_{GN} < g_{GN}^c$ , the system becomes Thirring like <sup>6</sup>.

We now argue that qualitatively the mass-generation phenomenon cannot be affected by the presence of supersymmetric partners of the fermion fields. Indeed, the only extra terms in the lagrangian (23) that could affect the dynamical mass generation are the terms mixing bosons and fermions,  $\bar{z}\partial_\mu z \bar{\Psi}\gamma^\mu\Psi$ . However, at the level of the effective action obtained from (23) by path-integrating out the  $z$  fields, the leading order contributions in a (low-energy) derivative expansion, are of order:  $\int d^3x \bar{\Psi}\gamma_\mu\Psi[(\partial^2 g^{\mu\nu} - \partial^\mu\partial^\nu)/m_Z^2]\bar{\Psi}\gamma_\nu\Psi$ . Such interactions constitute irrelevant operators in a renormalization group sense, even at large fermion flavour numbers  $N$ , and hence do not affect the fixed-point structure of the theory, responsible for mass generation, which is thus determined by the four-fermi terms <sup>7</sup>.

Within the context of dynamical mass generation, it is important to remark that in supersymmetric models dynamical mass generation can occur in a way compatible with unbroken supersymmetry only if the effective potential vanishes. This is a result of the equality of the fermion and boson masses,  $m_Z = m_f = m$ . In non-supersymmetric theories it is the minimization of the effective potential that selects the non-trivial solution of the Schwinger-Dyson (SD) analysis for the dynamical fermion mass. In contrast, as we shall argue below, in our supersymmetric case it is the *quantum* effective action, and *not* the effective potential, which is responsible for such a selection. The situation is similar to what happens in the two-dimensional supersymmetric  $O(3)$   $\sigma$ -model [33]. In that model, as a result of a constraint similar to (5), consistency among the supersymmetry Ward identities, obtained from the quantum effective action, selects the non-trivial solution for the dynamical masses, obtained from a SD analysis [34]. Below we shall not give the details, but we shall present the main arguments, which will be sufficient for our purposes in this letter. For simplicity we consider one “complex” superfield  $Z$  and work with its real components (14). The masses of the scalars  $\phi_i$ ,  $i = 1, 2$  and the Majorana spinors  $\chi_i$  are related by the Supersymmetry Ward identity:

$$\langle T\{\chi_i(x), \bar{\chi}_j(0)\} \rangle_o = (i\not{\partial} + m) \langle T\{\phi_i(x), \phi_j(0)\} \rangle_o \quad (30)$$

where  $\langle \dots \rangle_o$  denote correlators in the non-interacting theory.

On the other hand, it is known that the fields:

$$\left( -F_i = -\frac{\phi_i}{2}(\bar{\chi}_j\chi_j), \quad i\not{\partial}\chi_i, \quad \partial^2\phi_i \right) \quad (31)$$

constitute real superfields  $T_i$ , the kinetic multiplets of  $\Phi_i$ . Therefore, the vacuum expectation

---

<sup>6</sup>In this latter case we should point out that the non-trivial ultraviolet fixed point, found in the numerical studies of [32], might be related - under some sort of ultraviolet-infrared duality - to the non-trivial infrared fixed point of the three-dimensional  $QED$ , argued in [16].

<sup>7</sup>We note that, in a large-flavour-number,  $N$ , treatment, these four-fermi operators become renormalizable, thereby leading to non-trivial ultraviolet fixed-point structures [31, 28, 29].

values of the components of the superfield:

$$\phi_i T_i = \left( -\phi_i F_i, \quad \chi_i F_i - i\phi_i \not{\partial} \chi_i, \quad -\phi_i \partial^2 \phi_i + i\bar{\chi}_i \not{\partial} \chi_i + F_i F_i \right) \quad (32)$$

will be related by the supersymmetry Ward identities.

Using the equations of motion and the constraint (22) this superfield can be written as:

$$\phi_i T_i = \left( \nu, \quad \lambda, \quad \alpha - \nu^2 \right) \quad (33)$$

where

$$\begin{aligned} \nu &= -\frac{1}{2} \bar{\chi}_i \chi_i \\ \lambda &= -i\phi_i \not{\partial} \chi_i \\ \alpha &= \partial_\mu \phi_i \partial^\mu \phi_i \end{aligned} \quad (34)$$

Then, the corresponding supersymmetry Ward identities become [33]:

$$\begin{aligned} S_\lambda(p) - (\not{p} - 2m) D_\nu(p^2) &= 0 \\ \not{p} S_\lambda(p) - D_\alpha(p^2) + 2m(\not{p} - 2m) D_\nu(p^2) &= 0 \end{aligned} \quad (35)$$

where  $D_\nu, D_\alpha$  are the two-point Green's functions of  $\nu$  and  $\alpha$  fields respectively, and  $S_\lambda$  is the corresponding spinorial Green's function of  $\lambda$ . Note that the equations of motion, obeyed by the Green's functions, have been used in deriving the identities above.

In the context of the pure Gross-Neveu model in three space-time dimensions, one can compute the effective propagators by extending the two-dimensional analysis of ref. [33], in a straightforward manner. For instructive purposes we shall derive explicitly the  $D_\alpha$  propagator, pertaining to the Lagrange multipliers  $\alpha(x)$  implementing the constraint (22). Expanding about the vacuum  $\langle \alpha(x) \rangle = m^2$ ,  $\alpha(x) = \langle \alpha(x) \rangle + \alpha'(x)$ , and performing the  $z$  integration one arrives at an effective action

$$S_{eff,\alpha} = \int d^3x \text{Tr} \ln[\partial^2 + m^2 + \alpha'(x)] \quad (36)$$

The quadratic term in  $\alpha'(x)$  determines the effective propagator  $D_\alpha$  of the quantum field  $\alpha'$ . Passing onto a Fourier space one obtains:

$$S_{eff,\alpha}^{(2)} \sim \int \frac{d^3p}{(2\pi)^3} \int \frac{d^3k}{(2\pi)^3} \frac{1}{(k+p)^2 - m^2} \tilde{\alpha}'(-p) \frac{1}{k^2 - m^2} \tilde{\alpha}'(p) \quad (37)$$

where  $m$  is the dynamically-generated mass for (both) scalars and fermions (due to supersymmetry). From this, the propagator  $D_\alpha(p)$  is obtained immediately. Its  $p = 0$  limit is given by:

$$D_\alpha^{-1}(0) \sim \int d^3k \frac{1}{(k^2 - m^2)^2} \sim 1/m \quad (38)$$

In a similar manner one determines the rest of the Green's functions appearing in (35). For the Green's function  $D_\nu$ , associated with the linearized Gross-Neveu interactions, we note that the quantum corrections have been calculated in ref. [31], where a non-trivial ultraviolet fixed point structure has been revealed in a large-fermion-flavour number,  $N$ , framework.

With these in mind, one obtains the following results for the pertinent Green's functions, to leading order in  $1/N$  expansion:

$$\begin{aligned} D_\alpha^{-1}(0) &\sim \frac{1}{m} + \mathcal{O}\left(\frac{1}{N}\right) \\ D_\nu(0)^{-1} &\sim m \left(1 + \mathcal{O}\left[\frac{1}{N} \ln(\Lambda/m)\right]\right) \\ S_\lambda^{-1}(0) &\sim \int \frac{d^3k}{(2\pi)^3} \frac{1}{(k^2 - m^2)(\not{k} - m)} + \mathcal{O}\left(\frac{1}{N}\right) \sim \text{non-zero const} + \mathcal{O}\left(\frac{1}{N}\right) \end{aligned} \quad (39)$$

where  $\Lambda$  is an ultraviolet cut-off mass. In the above formulas factors of the dimensionful coupling constant  $g_1^2$  are understood where appropriate. Moreover, for our purposes in this work the detailed form of the  $\mathcal{O}(\frac{1}{N})$  corrections will not be important.

From the Ward identities (35), on the other hand, one has:

$$\begin{aligned} 4m^2 D_\nu(0) &= -D_\alpha(0) \\ 2m D_\nu(0) &= -S_\lambda(0) \end{aligned} \quad (40)$$

Then, on account of (39), we see that the first of the identities (40) is satisfied identically to this order in  $1/N$ , but one cannot exclude the trivial solution  $m = 0$ . Such an exclusion comes from the second of the identities (40), due to the structure of  $S_\lambda$ . The so-selected non-trivial solution for  $m$ , must be the one satisfying the SD equations [31], by consistency. A non-trivial verification of this will come by including the subleading  $1/N$  corrections. The reader should keep note of the crucial rôle of the  $CP^1$  constraint (22) in the above selection of the non-trivial SD mass gap by the quantum effective action of the  $N = 1$ -supersymmetric model [33, 34].

In the context of our model, involving both Gross-Neveu and Thirring interactions, a similar analysis goes through, with complexities coming from the non-linear realization of supersymmetry, and the new interactions in (23). Such deviations from the pure Gross-Neveu case, however, are in favour of the necessity of a non-zero mass gap, in order to fulfill the supersymmetry Ward identities. A detailed analysis along the above lines will be presented in a forthcoming publication. For the purposes of this note we restrict ourselves only to pointing out some subtleties, associated with the anomalous breaking of the fermion number in our model (c.f. figure 1). Indeed, after the  $z$  integration, and the implementation of the constraint as above, there are extra terms coupling fermions and  $\alpha$  multiplier fields in the effective action. One of them involves the divergence of the fermion current (after appropriate partial integrations in the action):

$$S_{eff}^{\chi, \alpha} \ni - \int d^3x \text{Tr} \frac{\partial_\mu (\bar{\chi}_i \gamma^\mu \chi_i)}{(\partial^2 + m^2)^2} \alpha(x) \quad (41)$$

If the fermion current  $\bar{\chi}_i \gamma^\mu \chi_i$  was conserved, then the gauge Ward identity would imply decoupling of this term from the physical correlators. However, as we mentioned above, there are anomalies in the model, in the massive phase for the fermions, associated with one-loop graphs of figure 1 [25, 5, 1]. Such anomalous terms should be properly taken into account in a detailed analysis of dynamical mass generation in our supersymmetric model, but we do not expect them to affect the selection of the non-trivial solution of the SD equations, characterising the pure Gross-Neveu case, studied in detail above.

An important additional comment concerns the kind of the three-dimensional dynamically-generated mass. At present, this seems to depend crucially on the relative sign of the mass, between the fermion species. In our analysis above, we have used a single superfield  $Z$ , whilst in our  $SU(2)$  model there are two such superfields. The SD analysis can be extended in that case straightforwardly, but alone it cannot make a selection among the two possible signs of the mass for these two superfields. Since the four fermion theories are not vector like, one does not have at first sight a way of energetically selecting the parity-conserving mass configuration. However, as we mentioned previously, the fact that the low-energy integration of magnon fields makes the model equivalent to a (vector-like) gauge theory with fermionic matter, is suggestive of the exclusion of the parity violating mass, on the basis of the theorems of ref. [30].

## 5 Instanton Effects, Supersymmetry and Superconductivity

A final issue we would like to address concerns the *exactness of superconductivity* in the presence of *non perturbative* effects. In the context of the  $SU(2) \times U_S(1)$  theory [1], superconductivity is associated with the masslessness of the  $B_\mu^3$  gauge boson of the unbroken  $U(1)$  subgroup of the  $SU(2)$  group, in the massive fermion phase [1, 5]. We now remark that, due to the compactness of the pertinent gauge group, instanton configurations - which, in 2+1 dimensions, are like monopoles - may give the  $U(1)$  gauge boson a small mass. In the dilute-instanton-gas approximation, in non-supersymmetric theories, this mass is of order [35]:

$$m_{B_3} \simeq e^{-S_0} \quad (42)$$

where  $S_0$  is the one instanton action. Such a small mass would destroy the exactness of the model's superconductivity, as we remarked earlier.

We shall argue in this section that supersymmetry favours superconductivity, by further suppressing the instanton contributions to the  $B_\mu^3$  gauge boson mass, as compared to the non-supersymmetric case. To this end, we first recall that a dynamical gauge theory is obtained in our model by integrating out  $z$  and  $\Psi$  fields [24]. In a non-supersymmetric theory, upon coupling to external electromagnetism, such a procedure leads, in the massive fermion phase, to the standard London action for superconductivity [5]. In our case, this procedure leads to a supersymmetric gauge theory  $U(1) \times U_S(1)$ . Indeed, by integrating out  $z$  fields one obtains Maxwell kinetic terms for the gauge fields, in the phase where the magnon fields are massive [24].

In our supersymmetric theory, the Yukawa coupling of the gaugino  $\eta$  to  $\Psi$  and  $z$  fields results, after the  $z, \Psi$  integration in Majorana kinetic terms for  $\eta$ , as required by  $N = 1$  supersymmetry. This can be readily seen by a one loop computation, in analogy with the bosonic  $z$  part [24]. The relevant graphs, in the massive phase  $m_Z = m_f = m$  (due to supersymmetry), result in the following integral:

$$\gamma^\mu \int d^3k \frac{k_\mu + p_\mu}{[(k+p)^2 - m^2][k^2 - m^2]} \sim \frac{\not{p}}{2m} \quad (43)$$

yielding a Majorana kinetic term  $\frac{i}{2m} \bar{\eta} \not{\partial} \eta$  for the gaugino. One can easily verify the manifest  $N = 1$  supersymmetry between this term and the corresponding Maxwell terms  $-\frac{1}{4m} F_{\mu\nu}^2$ , obtained by the  $z$  and  $\Psi$  integration [24, 5].

We now remark that in three dimensional supersymmetric gauge theories it is known [26] that supersymmetry cannot be broken, due to the fact that the Witten index  $(-1)^F$ , where  $F$  is the fermion number, is always non zero. Thus, in supersymmetric theories the presence of instantons should give a small mass, if at all, in both the gauge boson and the associated gaugino. Although at present there is no rigorous proof of this fact, however, the arguments of ref. [26] indicated that the resulting masses will be even more suppressed than the corresponding ones in the non supersymmetric case,

$$m_{B_3} = m_\eta = e^{-2S_0} \quad (44)$$

with  $S_0$  the one-instanton action.

We should point out, however, that there is an alternative scenario [26], which could be in operation in our superconducting model. It is possible that supersymmetry is broken by having the system in a ‘false’ vacuum, where the gauge boson remains massless, even in the presence of non perturbative configurations, while the gaugino acquires a small mass, through non perturbative effects. The life time, however, of this false vacuum is very long [26], and hence superconductivity can occur, in the sense that the system will remain in that false vacuum for a very long period of time, longer than any other time scale in the problem.

Whichever of the two scenaria is realized in the model, from a condensed-matter point of view the important conclusion, obtained from the above analysis, is that the coupling of the superconducting planes due to interlayer electron hopping, associated with the presence of the gaugino field  $\eta$ , helps stabilizing superconductivity, which otherwise would be jeopardized by non-perturbative effects.

## 6 Discussion

In this work we have demonstrated the possibility of  $N=1$  Supersymmetric gauge theories in the context of a spin-charge separation ansatz of the  $SU(2) \times U_S(1)$  gauge model of [1]. Such models may be relevant for the physics of superconducting gaps which open up at the nodes of

a d-wave gap of high- $T_c$  cuprates. The supersymmetry was achieved without introducing unphysical degrees of freedom. However it necessitates the coupling of the superconducting planes. Its presence, seems to suppress the effects of instantons of the gauge field  $SU(2)$ , which could jeopardise superconductivity in the model. As far as the lattice system is concerned the supersymmetry is achieved *modulo* irrelevant operators in a renormalization-group sense. This may imply that our considerations in this work might also be relevant to the construction of more general supersymmetric gauge theories on the lattice, in the sense of obtaining supersymmetric continuum theories by dropping possibly non-supersymmetric, renormalization-group irrelevant operators.

We believe that our work may prove useful towards an exact discussion of phase diagrams of three-dimensional effective gauge models of antiferromagnetic superconductors, via the analysis of the quantum moduli space of gauge theories, in the spirit of Seiberg and Witten [14]. In this respect, we note that  $N = 1$  supersymmetry in three dimensions, which we have considered here as the minimal way of supersymmetrization of a doped spin-charge separated antiferromagnet, without the introduction of extra degrees of freedom, cannot yield exact results. It is the  $N = 4$  supersymmetry in three-dimensional gauge theories which can produce such results. In three dimensions,  $N = 2$  theories may also allow for some exact results, in connection with the geometry of their quantum moduli space [15].

At present, our physical understanding for a condensed-matter spin-charge separated model exhibiting  $N = 2$  supersymmetry is not complete. One might speculate that, since  $N = 2$  three-dimensional supersymmetric theories are obtained [15] by dimensional reduction of four-dimensional  $N = 1$  supersymmetric theories, such models might have some relevance to a possible extension of the ideas in our work beyond the planar structures. We shall present a more detailed study of such models in a future work [36]. We should remark however that, as far as the spinon and holon degrees of freedom are concerned, the extension to  $N = 2$  supersymmetry is immediate, with no extra doubling of degrees of freedom. The novel feature, compared to the  $N=1$  case, is the presence of a *Dirac-like* gaugino. Due to its Dirac nature, the gaugino may now carry non-trivial charge under the external electromagnetism, and thus the effective action conserves the electric charge, in contrast to the present situation with a Majorana gaugino. In view of the aforementioned embedding of 3-dimensional  $N=2$  theories in 4-dimensional  $N=1$  supersymmetric theories, this possibility of conservation of the electric charge may be related to the exact conservation of electric charge in four dimensional space times. From the point of view of dynamical mass generation, we should remark that, at first sight, the  $N = 2$  3-dimensional models appear not to generate a dynamical mass. This is due to the fact that such theories are obtained from  $N = 1$  4-dimensional models, where claims have been made [37] that non-renormalization theorems in the supersymmetric SD equations yield only the trivial solution for the mass. However, such claims have been questioned recently [38]. From our point of view we consider the issue as still open.

Another comment we would like to make concerns the fate of supersymmetry at finite temperatures. We expect the supersymmetry to be broken at finite temperatures, which results in different masses for spinon and holons, a situation probably met in realistic cases. However,



even in such a case of broken supersymmetry, the existence of a supersymmetric vacuum at zero temperatures is useful in providing some exact information about the phase diagram along the lines mentioned above.

Before closing we should also stress that our results apply even to one-dimensional chains of holons, which may characterize certain underdoped cuprates in the so-called stripe phase [39]. Such systems appear to be described by a spin-charge separated phase, where the holon degrees of freedom lie on one-space dimensional stripes (chains), spatially separated by regions of zero doping. As discussed in ref. [27], spin charge separation in one (spatial) dimensional antiferromagnetic models leads to gauge theories of Dirac fermions coupled to a  $CP^1$   $\sigma$ -model. The continuum action is similar in form, but in two space-time dimensions, with the action (4). In such a case, the resulting  $N = 1$  supersymmetric extension will again involve combined Gross-Neveu and Thirring interactions. Such (1+1)-dimensional models have been studied previously in the literature [33]. As far as supersymmetry and dynamical mass generation are concerned, such models share the same qualitative features as their  $(2 + 1)$ -dimensional counterparts, discussed here. The gauginos in such one-dimensional theories could then describe (effective) electron hopping across the chains. At present we are agnostic as to whether such supersymmetric spin-charge separated models play any crucial rôle on the physics of the stripe phase of the underdoped cuprates.

## Acknowledgements

G.A.D. and B.C.G. wish to thank the Department of (Theoretical) Physics of Oxford University for the hospitality.

## References

- [1] K. Farakos and N.E. Mavromatos, preprint cond-mat/9611072, to appear in *Phys. Rev. B*; preprint hep-lat/9707027, to appear in *Mod. Phys. Lett. A*.
- [2] I. Affleck, Z. Zou, T. Hsu and P.W. Anderson, *Phys. Rev. B* **39** (1989), 11538.
- [3] P. W. Anderson, *Science* **235** (1987), 1196.
- [4] R.B. Laughlin, Proc. 4th Chia meeting on *Common Trends in Condensed Matter and Particle Physics*, Chia-Laguna (Italy), September 1994.
- [5] N. Dorey and N. E. Mavromatos, *Phys. Lett. B* **250** (1990), 107; *Nucl. Phys. B* **386** (1992), 614;  
For a comprehensive review of this approach see: N. Mavromatos, *Nucl. Phys. B (Proc. Suppl.)* **33C** (1993), 145.

- [6] C.C. Tsuei *et al.*, *Phys. Rev. Lett.* **73** (1994), 593;  
K.A. Moler *et al.*, *Phys. Rev. Lett.* **73** (1994), 2744;  
D.A. Bonn *et al.*, *ibid.* **68** (1992), 2390.
- [7] K. Krishana *et. al.*, *Science* **277** (1997), 83.
- [8] R. Movshovich *et al.*, preprint cond-mat/9709061.
- [9] R.B. Laughlin, preprint cond-mat/9709004.
- [10] A.V. Balatsky, preprint cond-mat/9710323.
- [11] J. Fröhlich and P. Marchetti, *Phys. Rev.* **B46** (1992), 6535;  
J. Fröhlich, T. Kerler and P. Marchetti, *Nucl. Phys.* **B374** (1992), 511.
- [12] P.B. Wiegmann, *Phys. Rev. Lett.* **60** (1988), 821;  
S. Sarkar, *J. Phys.* **A23** (1990) L409; *J. Phys.* **A24** (1991), 1137;  
F.H.L. Essler, V.A. Korepin and K. Schoutens, *Phys. Rev. Lett.* **68** (1992), 2960 ;
- [13] A. Lerda and S. Sciuto, *Nucl. Phys.* **B410** (1993), 577.
- [14] N. Seiberg, *Phys. Lett* **B206** (1988), 75;  
N. Seiberg and E. Witten, *Nucl. Phys.* **B406** (1994), 19.
- [15] O. Aharony, A. Hanany, K. Intriligator, N. Seiberg and M. Strassler, hep-th/9703310, and references therein.
- [16] I.J. R. Aitchison and N.E. Mavromatos, *Phys. Rev.* **B53** (1996), 9321; I.J. R. Aitchison, G. Amelino-Camelia, M. Klein-Kreisler, N.E. Mavromatos and D. Mc. Neill, *Phys. Rev.* **B56** (1997), in press.
- [17] N. Dorey and N. E. Mavromatos, *Phys. Rev.* **B44** (1991), 5286.
- [18] R. D. Pisarski, *Phys. Rev.* **D29** (1984), 2423;  
T. W. Appelquist, M. Bowick, D. Karabali and L. C. R. Wijewardhana, *Phys. Rev.* **D33** (1986), 3704.  
T. W. Appelquist, D. Nash and L. C. R. Wijewardhana, *Phys. Rev. Lett.* **60** (1988), 2575.
- [19] E. Dagotto, A. Kocic and J.B. Kogut, *Phys. Rev. Lett.* **62** (1989), 1083; *Nucl. Phys.* **B334** (1990), 279.
- [20] K. Farakos and G. Koutsoumbas, *Phys. Lett.* **B178** (1986), 260.

- [21] P. Maris, *Phys. Rev.* **D54** (1996), 4049.
- [22] K. Farakos, G. Koutsoumbas and G. Zoupanos, *Phys. Lett.* **B249** (1990), 101.
- [23] N. E. Mavromatos and M. Ruiz-Altaba, *Phys. Lett.* **A142** (1989), 419.
- [24] A.M. Polyakov, *Gauge Fields and Strings* (Harwood 1987);  
S. Deser and A.N. Redlich, *Phys. Rev. Lett.* **61** (1989), 1541.
- [25] A. Kovner and B. Rosenstein, *Phys. Rev.* **B42** (1990), 4748; the absence of a local order parameter in the gauge symmetry breaking patterns of (2+1)-dimensional QED is reminiscent of, although not identical to, the ‘topological order/disorder transition’ in planar systems proposed by J. Kosterlitz and D. Thouless, *J. Phys.* **C6** (1973), 1181.
- [26] I. Affleck, J. Harvey and E. Witten, *Nucl. Phys.* **B206** (1982), 413.
- [27] R. Shankar, *Phys. Rev. Lett.* **63** (1989), 203; *Nucl. Phys.* **B330** (1990), 433.
- [28] T. Kim, W. Kye and J. Kim, *Phys. Rev.* **D52** (1995), 6109.
- [29] T. Dateki, *Prog. Theor. Phys.* **97** (1997), 921; hep-th/9701183.
- [30] C. Vafa and E. Witten, *Comm. Math. Phys.* **95** (1984), 257.
- [31] S. Hands, A. Kocic and J.B. Kogut, *Phys. Lett.* **B273** (1991), 111;  
H. He, Y. Kuang, Q. Wang, and Y. Yi, *Phys. Rev.* **D45** (1992), 4610.
- [32] L. Del Debbio, S. Hands and J.C. Mehegan, *Nucl. Phys.* **B502** (1997), 269.
- [33] O. Alvarez, *Phys. Rev.* **17** (1978), 1123.
- [34] R. D. Pisarski, *Phys. Rev.* **D29** (1984), 2423.
- [35] A. M. Polyakov, *Nucl. Phys.* **B120** (1977), 429.
- [36] G.A. Diamandis, B.C. Georgalas, A.B. Lahanas and N.E. Mavromatos, work in progress.
- [37] T.E. Clark and S.T. Love, *Nucl. Phys.* **B310** (1988), 371.
- [38] T. Appelquist, A. Nyffeler and S.B. Selipsky, preprint hep-th/9709177;  
A. Kaiser and S.B. Selipsky, preprint hep-th/9708087.
- [39] J. Tranquada *et al.*, *Nature* **375** (1995), 561; *Phys. Rev.* **B54** (1996), 7489;  
O. Zachar, S.A. Kivelson and V.J. Emery, preprint cond-mat/9702055, and references therein.